2. Scheme bloc structurale

2.1. Preliminarii

Modelul matematic:

Analiza – Se împarte sistemul în *elemente* simple;
urmează studiul lor separat,
descrierea matematică şi
evidenţierea ***cauzelor*** şi ***efectelor***.

Se asociază fiecărui *element* o schemă bloc parţială.

Sinteza – Cf. conexiunilor între elem. sist., se înlănţuie schemele bloc parţiale şi se obţine ***schema bloc structurală*** a sistemului.

Exemplul 2.1.
Motor electric de cc
Fenomene: electrice,
electromagnetice,
electromecanice.

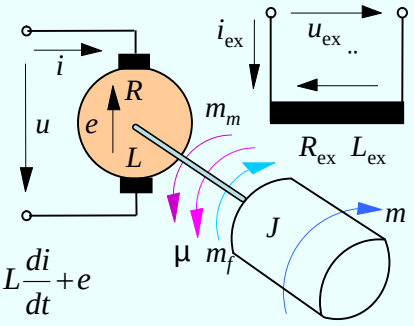


Fig. II.10

Analiza

(a) Circuitul rotoric: $u = Ri + L\frac{di}{dt} + e$

(b) Circuitul rotoric (tcem): $e = k_1\mu$

(c) Rotorul (mişc. rotaţie): $J\frac{d\mu}{dt} = m_m - m_f - m$

(d) Rotorul (cuplul elmag.): $m_m = k_2 i$

(e) Rotorul (cuplul de frec.): $m_f = k_3\mu$, k_1, k_2, k_3 – constante

- Se explicitează relaţia de cauzalitate.
- Se aplică transformarea Laplace.

ECUAŢII — CAUZA ➡ EFECT — INTRARE – IEŞIRE

ECUAŢII	CAUZA ➡ EFECT	INTRARE – IEŞIRE
$u = Ri + L\frac{di}{dt} + e$	$u - e \Rightarrow i$	$I(s) = \frac{1}{Ls+R}[U(s) - E(s)]$ (a)
$e = k_1 \mu$	$\mu \Rightarrow e$	$E(s) = k_1 \Omega(s)$ (b)
$J\frac{d\mu}{dt} = m_m - m_f - m$	$\left.\begin{matrix} m_m - \\ m_f - \\ m \end{matrix}\right\} \Rightarrow \mu$	$\Omega(s) = \frac{1}{Js}[M_m(s) - M_f(s) - M(s)]$ (c)
$m_m = k_2 i$	$i \Rightarrow m_m$	$M_m(s) = k_2 I(s)$ (d)
$m_f = k_3 \mu$	$\mu \Rightarrow m_f$	$M_f(s) = k_3 \Omega(s)$ (e).

INTRARE ➡ IEŞIRE — SCHEME BLOC PARŢIALE

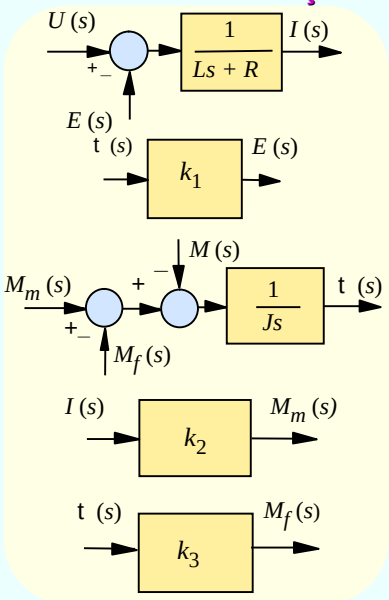
$$I(s) = \frac{1}{Ls+R}[U(s) - E(s)] \text{ (a)}$$

$$E(s) = k_1 \Omega(s) \text{ (b)},$$

$$\Omega(s) = \frac{1}{Js}[M_m(s) - M_f(s) - M(s)] \text{ (c)}$$

$$M_m(s) = k_2 I(s) \text{ (d)}.$$

$$M_f(s) = k_3 \Omega(s) \text{ (e)}.$$



$$I(s)=\frac{1}{Ls+R}[U(s)-E(s)](a) \quad E(s)=k_1\Omega(s) \quad (b) \quad \Omega(s)=\frac{1}{Js}[M_m(s)-M_f(s)-M(s)](c)$$

$$M_m(s)=k_2 I(s)(d) \qquad M_f(s)=k_3\Omega(s)(e).$$

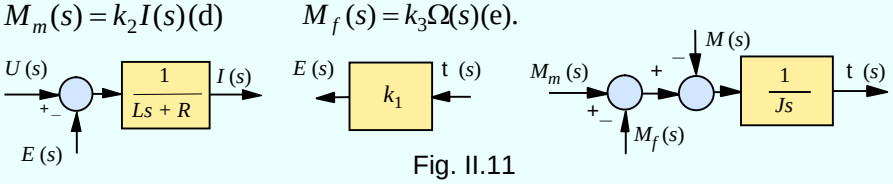
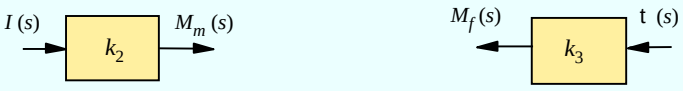


Fig. II.11



Sinteza

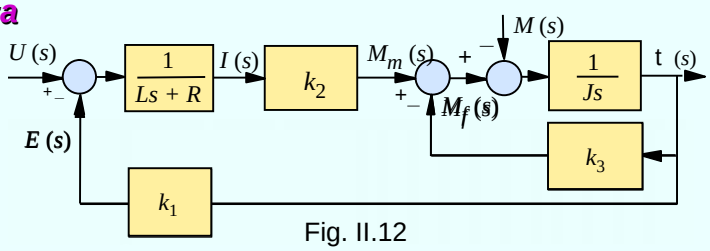


Fig. II.12

Acţionări electrice cu turaţie reglabilă: $u \Rightarrow \mu$, $m \Rightarrow \mu$

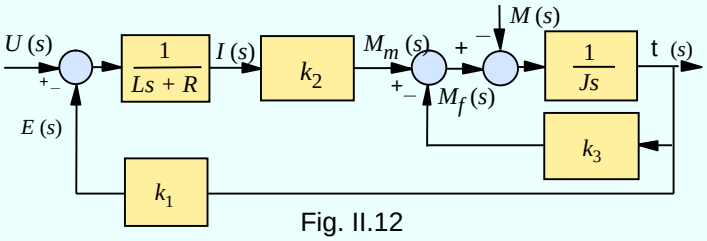
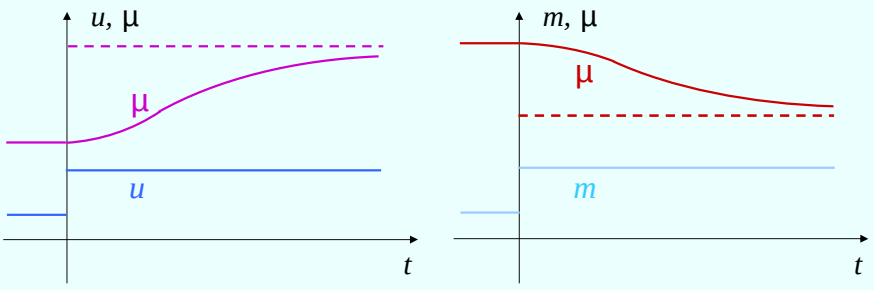
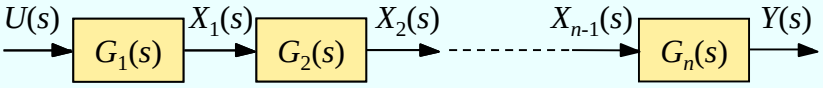


Fig. II.12



2.2. Conexiuni elementare

1î Conexiunea «serie»



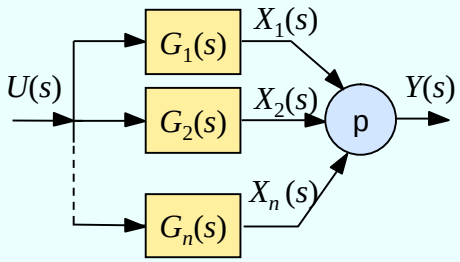
$$X_i(s) = G_i(s)X_{i-1}(s), \quad i = \overline{1,n}, \qquad (2.1)$$

$$X_0(s) = U(s), \quad X_n(s) = Y(s). \qquad (2.2)$$

$$Y(s) = G(s)U(s), \quad G(s) = \prod_{i=1}^{n} G_i(s). \qquad (2.3)$$

Funcţia de transfer echivalentă = produsul funcţiilor de transfer.

2î Conexiunea «paralel»



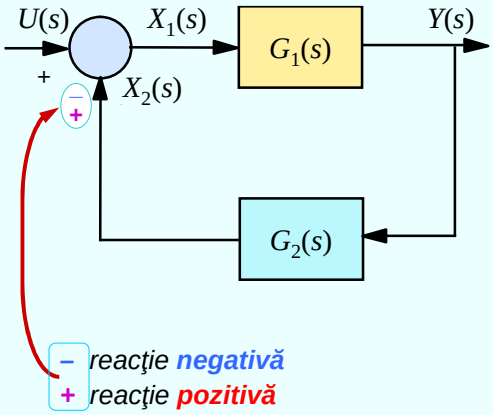
$$X_i(s) = G_i(s)U(s), \quad i = \overline{1,n}, \qquad (2.4)$$

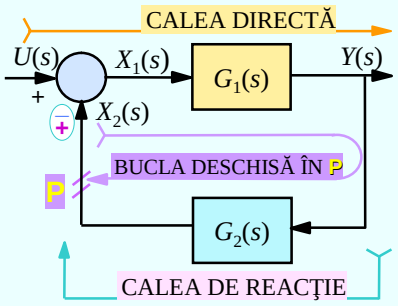
$$Y(s) = \sum_{i=1}^{n} X_i(s). \qquad (2.5)$$

$$Y(s) = G(s)U(s), \quad G(s) = \sum_{i=1}^{n} G_i(s). \qquad (2.6)$$

Funcţia de transfer echivalentă = suma funcţiilor de transfer.

3î Conexiunea «cu reacţie»





$$X_1(s) = U(s) \mp X_2(s) \quad (2.7)$$

$$Y(s) = G_1(s)X_1(s) \quad (2.8)$$

$$X_2(s) = G_2(s)Y(s). \quad (2.9)$$

$$Y(s) = G_1(s)\left[U(s) \mp G_2(s)Y(s)\right],$$

$$\left[1 \pm G_1(s)G_2(s)\right]Y(s) = G_1(s)U(s), \quad (2.11)$$

\+ pentru ***reacţie negativă***,

− pentru ***reacţie pozitivă***.

$$Y(s) = G_0(s)U(s),\ G_0(s) = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)}, \quad (2.10)$$

Funcţia de transfer echivalentă = raportul dintre funcţia de transfer a căii directe şi 1 ± funcţia de transfer a buclei deschise în punctul P .

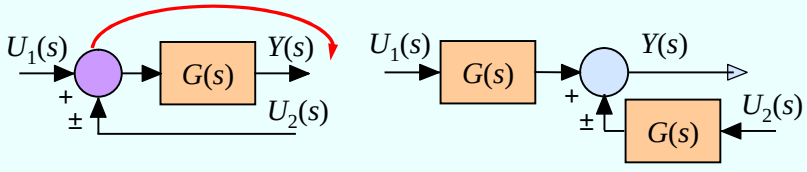
2.3. Transfigurarea schemelor bloc structurale

- Analiza şi sinteza sistemelor dinamice reclamă determinarea relaţiilor dintre două sau mai multe mărimi ale schemei bloc structurale.
- Prin operaţii de ***transfigurare*** se obţin rezultatele căutate.
- Ele se execută conform unor ***identităţi de transfigurare***.

Fig. II. 17. Identităţi de transfigurare

4î Deplasarea unui sumator de la intrarea la ieşirea unui bloc

a (schema iniţială) b (schema finală)



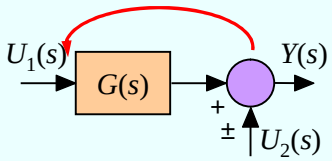
(2.12) $Y(s) = G(s)[U_1(s) \pm U_2(s)]$

(2.13) $Y(s) = G(s)U_1(s) \pm G(s)U_2(s)$

Fig. II. 17. Identităţi de transfigurare (continuare)

5î Deplasarea unui sumator de la ieşirea la intrarea unui bloc

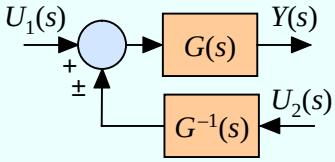
a (schema iniţială)



(2.14)

$$Y(s) = G(s)U_1(s) \pm U_2(s)$$

b (schema finală)



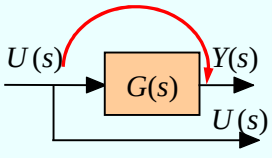
(2.15)

$$Y(s) = G(s)[U_1(s) \pm G^{-1}(s)U_2(s)]$$

Fig. II. 17. Identităţi de transfigurare (continuare)

6î Deplasarea unui punct de ramificare de la intrare la ieşire

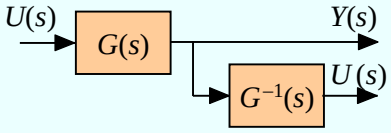
a (schema iniţială)



(2.16)

$$Y(s) = G(s)U(s),$$
$$U(s) = U(s)$$

b (schema finală)



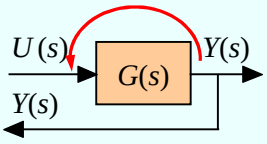
(2.17)

$$Y(s) = G(s)U(s),$$
$$U(s) = G^{-1}(s)G(s)U(s)$$

Fig. II. 17. Identităţi de transfigurare (continuare)

7î Deplasarea unui punct de ramificare de la ieşire la intrare

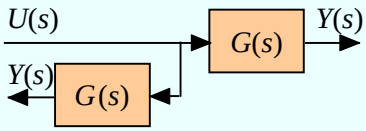
a (schema iniţială)



(2.18)

$$Y(s) = G(s)U(s),$$
$$Y(s) = Y(s)$$

b (schema finală)



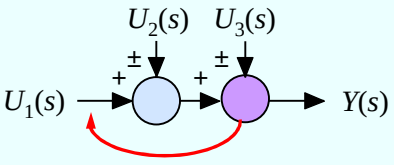
(2.19)

$$Y(s) = G(s)U(s),$$
$$Y(s) = G(s)U(s)$$

Fig. II. 17. Identităţi de transfigurare (continuare)

8î Comutativitatea sumatoarelor

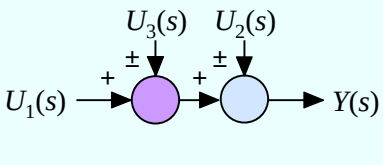
a (schema iniţială)



(2.20)

$$Y(s) = [U_1(s) \pm U_2(s)] \pm U_3(s)$$

b (schema finală)



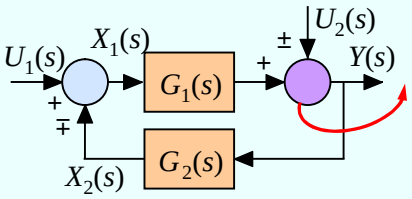
(2.21)

$$Y(s) = [U_1(s) \pm U_3(s)] \pm U_2(s)$$

Fig. II. 17. Identităţi de transfigurare (continuare)

9î Deplasarea sumatorului din interiorul conexiunii <<cu reacţie>> la ieşire

a (schema iniţială)



$$X_1(s) = U_1(s) \mp X_2(s)$$

$$X_2(s) = G_2(s)Y(s).$$

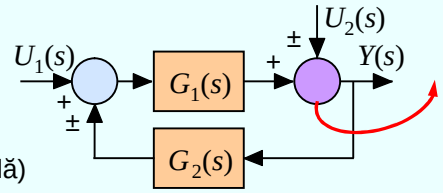
$$Y(s) = G_1(s)X_1(s) \pm U_2(s)$$

$$Y(s) = G_1(s)[U_1(s) \mp G_2(s)Y(s)] \pm U_2(s)$$

$$[1 \pm G_1(s)G_2(s)]Y(s) = G_1(s)U_1(s) \pm U_2(s)$$

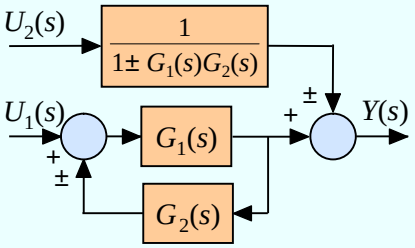
$$Y(s) = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)}U_1(s) \pm \frac{1}{1 \pm G_1(s)G_2(s)}U_2(s),$$

9î Deplasarea sumatorului (continuare)



a (schema iniţială)

$$Y(s) = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)}U_1(s) \pm \frac{1}{1 \pm G_1(s)G_2(s)}U_2(s),$$

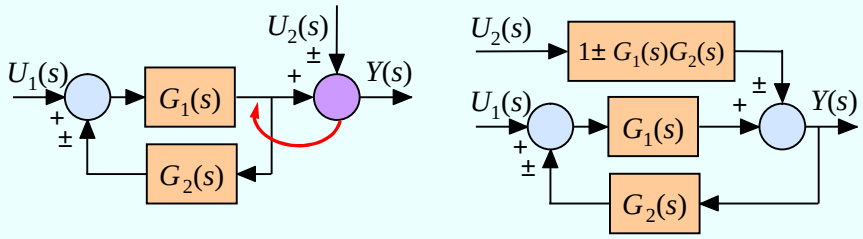


b (schema finală)

Fig. II. 17. Identităţi de transfigurare (continuare)

10î Deplasarea sumatorului de la ieşire în interiorul conexiunii <<cu reacţie>>

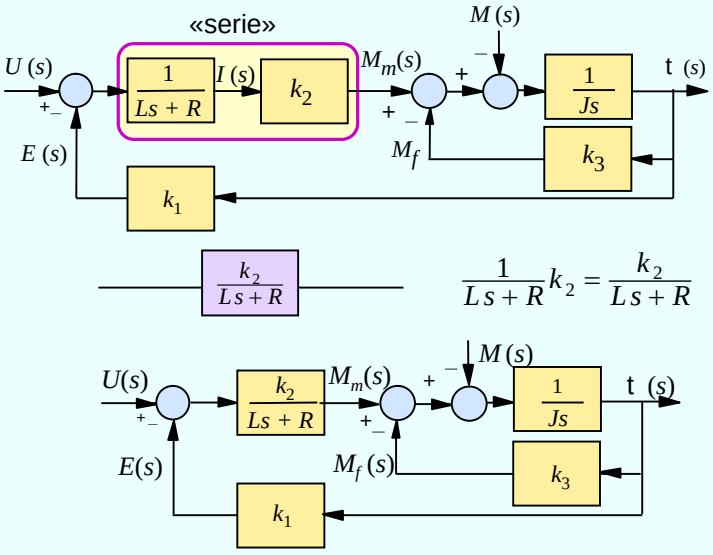
a (schema iniţială) b (schema finală)



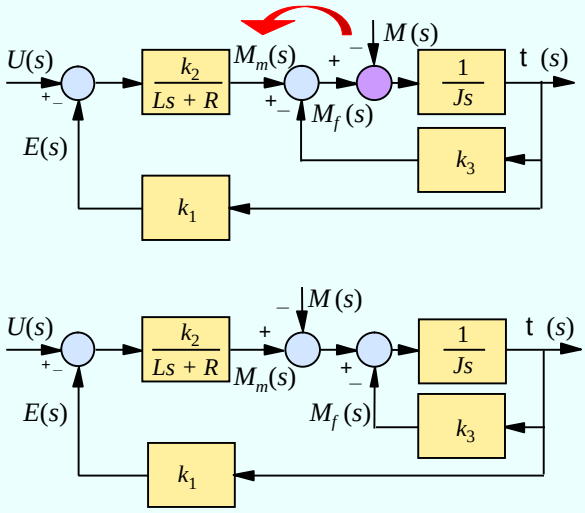
$$Y(s)=\frac{G_1(s)}{1\pm G_1(s)G_2(s)}U_1(s)\pm U_2(s),$$

$$Y(s)=\frac{G_1(s)}{1\pm G_1(s)G_2(s)}U_1(s)\pm\frac{1}{1\pm G_1(s)G_2(s)}[1\pm G_1(s)G_2(s)]U_2(s).$$

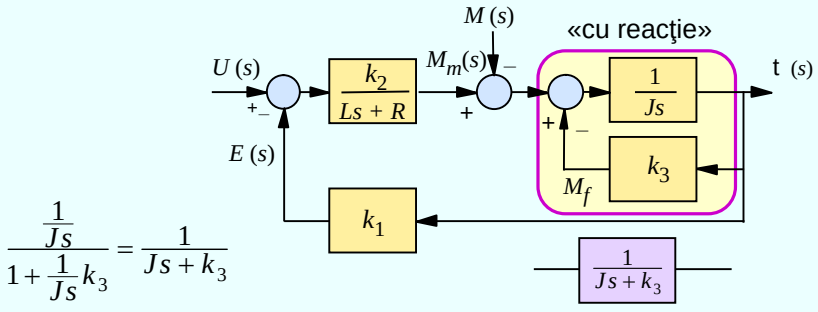
Exemplul 2.2. Motor electric de cc (continuare)



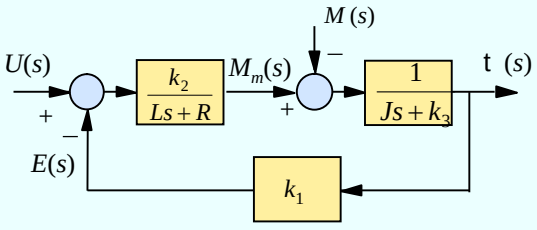
Exemplul 2.2. Motor electric de cc



Exemplul 2.2. Motor electric de cc (continuare)



$$\frac{\frac{1}{Js}}{1+\frac{1}{Js}k_3}=\frac{1}{Js+k_3}$$



Exemplul 2.3.
Motor electric de cc

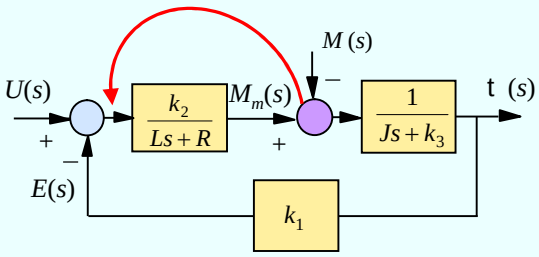
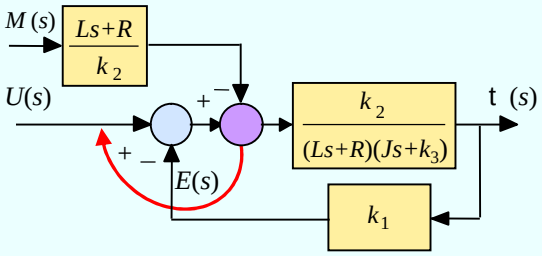


Fig. II.16



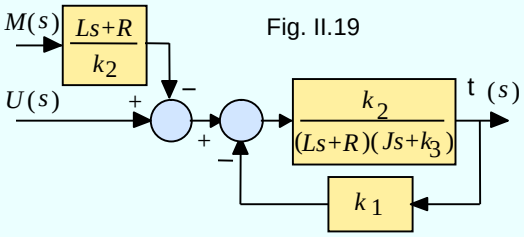


Fig. II.19

Exemplul 2.3.
Motor electric de cc
(continuare)

$$\Omega(s) = G_0(s)U(s) - G_{0m}(s)M(s),$$

$$G_0(s) = \frac{\dfrac{k_2}{(Ls+R)(Js+k_3)}}{1+\dfrac{k_1k_2}{(Ls+R)(Js+k_3)}} = \frac{k_2}{LJs^2+(RJ+Lk_3)s+Rk_3+k_1k_2}$$

$$G_{0m}(s) = \frac{Ls+R}{k_2}\,\frac{\dfrac{k_2}{(Ls+R)(Js+k_3)}}{1+\dfrac{k_1k_2}{(Ls+R)(Js+k_3)}} = \frac{Ls+R}{LJs^2+(RJ+Lk_3)s+Rk_3+k_1k_2} \quad . \blacksquare$$

Paşi de transfigurare:

- a) Se echivalează conexiunile «serie».
- b) Se echivalează conexiunile «paralel».
- c) Se echivalează conexiunile «cu reacţie».
- d) Se deplasează punctele de ramificare şi/sau sumatoarele conform identităţilor 4° – 8 °.
- e) Se repetă operaţiile de la paşii a – d până se obţine rezultatul dorit.

2.4. O schemă bloc operaţională asociată unei funcţii de transfer

$$Y(s)=\underbrace{\frac{b_m s^m+b_{m-1}s^{m-1}+...+b_0}{a_n s^n+a_{n-1}s^{n-1}+...+a_0}}_{G(s)}U(s), \qquad (1.10)$$

Observaţia 1.2 (v. 1.4)

Numărătorul modelează operaţii de amplificare şi derivare; are efect de ***anticipare***.

Numitorul modelează operaţii bazate pe integrare; are efect de ***întârziere***.

O schema bazată numai pe integratoare

Exemplu

$$Y(s)=\underbrace{\frac{b_3s^3+b_2s^2+b_1s+b_0}{a_3s^3+a_2s^2+a_1s+a_0}}_{G(s)}U(s);\quad a_3\neq 0 \text{ şi cel puţin un } b_j\neq 0.$$

Se elimina numitorul $a_3s^3+a_2s^2+a_1s+a_0$.

$$\left(a_3s^3+a_2s^2+a_1s+a_0\right)Y(s)=\left(b_3s^3+b_2s^2+b_1s+b_0\right)U(s), \qquad \times\frac{1}{a_3s^3}$$

$$\left(1+\frac{a_2}{a_3}\frac{1}{s}+\frac{a_1}{a_3}\frac{1}{s^2}+\frac{a_0}{a_3}\frac{1}{s^3}\right)Y(s)=\left(\frac{b_3}{a_3}+\frac{b_2}{a_3}\frac{1}{s}+\frac{b_1}{a_3}\frac{1}{s^2}+\frac{b_0}{a_3}\frac{1}{s^3}\right)U(s).$$

$$Y(s)+\frac{a_2}{a_3}\frac{1}{s}Y(s)+\frac{a_1}{a_3}\frac{1}{s^2}Y(s)+\frac{a_0}{a_3}\frac{1}{s^3}Y(s)=$$

$$=\frac{b_3}{a_3}U(s)+\frac{b_2}{a_3}\frac{1}{s}U(s)+\frac{b_1}{a_3}\frac{1}{s^2}U(s)+\frac{b_0}{a_3}\frac{1}{s^3}U(s).$$

$$Y(s)=\frac{b_3}{a_3}U(s)+\frac{b_2}{a_3}\frac{1}{s}U(s)-\frac{a_2}{a_3}\frac{1}{s}Y(s)+$$

$$+\frac{b_1}{a_3}\frac{1}{s^2}U(s)-\frac{a_1}{a_3}\frac{1}{s^2}Y(s)+\frac{b_0}{a_3}\frac{1}{s^3}U(s)-\frac{a_0}{a_3}\frac{1}{s^3}Y(s).$$

$$Y(s)=\frac{b_3}{a_3}U(s)+\frac{1}{s}\left(\frac{b_2}{a_3}U(s)-\frac{a_2}{a_3}Y(s)\right)+$$

$$+\frac{1}{s^2}\left(\frac{b_1}{a_3}U(s)-\frac{a_1}{a_3}Y(s)\right)+\frac{1}{s^3}\left(\frac{b_0}{a_3}U(s)-\frac{a_0}{a_3}Y(s)\right)$$

$$Y(s)=\frac{b_3}{a_3}U(s)+\frac{1}{s}\left(\frac{b_2}{a_3}U(s)-\frac{a_2}{a_3}Y(s)\right)+$$

$$+\frac{1}{s^2}\left(\frac{b_1}{a_3}U(s)-\frac{a_1}{a_3}Y(s)+\frac{1}{s}\left(\frac{b_0}{a_3}U(s)-\frac{a_0}{a_3}Y(s)\right)\right),$$

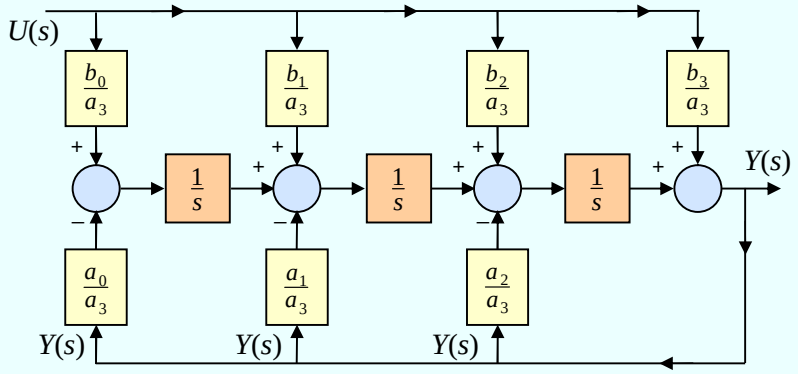
$$Y(s)=\frac{b_3}{a_3}U(s)+\frac{1}{s}\left(\frac{b_2}{a_3}U(s)-\frac{a_2}{a_3}Y(s)\right)+$$

$$+\frac{1}{s}\frac{1}{s}\left(\frac{b_1}{a_3}U(s)-\frac{a_1}{a_3}Y(s)+\frac{1}{s}\left(\frac{b_0}{a_3}U(s)-\frac{a_0}{a_3}Y(s)\right)\right),$$

$$Y(s)=\frac{b_3}{a_3}U(s)+\frac{1}{s}\left(\frac{b_2}{a_3}U(s)-\frac{a_2}{a_3}Y(s)+\right.$$

$$\left.+\frac{1}{s}\left(\frac{b_1}{a_3}U(s)-\frac{a_1}{a_3}Y(s)+\frac{1}{s}\left(\frac{b_0}{a_3}U(s)-\frac{a_0}{a_3}Y(s)\right)\right)\right),$$

$$Y(s)=\frac{b_3}{a_3}U(s)+\frac{1}{s}\left(\frac{b_2}{a_3}U(s)-\frac{a_2}{a_3}Y(s)+\frac{1}{s}\left(\frac{b_1}{a_3}U(s)-\frac{a_1}{a_3}Y(s)+\frac{1}{s}\left(\frac{b_0}{a_3}U(s)-\frac{a_0}{a_3}Y(s)\right)\right)\right).$$



Cazul general

$$Y(s)=\frac{b_m s^n+b_{n-1}s^{n-1}+\ldots+b_0}{a_m s^n+a_{n-1}s^{n-1}+\ldots+a_0}U(s),\ a_n\neq 0 \quad \text{şi cel puţin un } b_j\neq 0.$$

$$\left(a_m s^n+a_{n-1}s^{n-1}+\ldots+a_0\right)Y(s)=\left(b_m s^m+b_{m-1}s^{m-1}+\ldots+b_0\right)U(s) \qquad \times\frac{1}{a_n s^n}$$

$$Y(s)=\frac{b_n}{a_n}U(s)+\frac{b_{n-1}}{a_n}\frac{1}{s}U(s)-\frac{a_{n-1}}{a_n}\frac{1}{s}Y(s)+$$

$$+\frac{b_1}{a_n}\frac{1}{s^{n-1}}U(s)-\frac{a_1}{a_n}\frac{1}{s^{n-1}}Y(s)+\frac{b_0}{a_n}\frac{1}{s^n}U(s)-\frac{a_0}{a_n}\frac{1}{s^n}Y(s).$$

$$Y(s)=\frac{b_n}{a_n}U(s)+\frac{1}{s}\left(\frac{b_{n-1}}{a_n}U(s)-\frac{a_{n-1}}{a_n}Y(s)\right)+\ldots+$$

$$+\frac{1}{s^{n-1}}\left(\frac{b_1}{a_n}U(s)-\frac{a_1}{a_n}Y(s)\right)+\frac{1}{s^n}\left(\frac{b_0}{a_n}U(s)-\frac{a_0}{a_n}Y(s)\right)$$

$$Y(s)=\frac{b_n}{a_n}U(s)+\frac{1}{s}\left(\frac{b_{n-1}}{a_n}U(s)-\frac{a_{n-1}}{a_n}Y(s)+\ldots+\right.$$

$$\left.+\frac{1}{s}\left(\frac{b_1}{a_n}U(s)-\frac{a_1}{a_n}Y(s)+\frac{1}{s}\left(\frac{b_0}{a_n}U(s)-\frac{a_0}{a_n}Y(s)\right)\underbrace{\ldots}_{n}\right)\right)$$

$$Y(s)=\frac{b_n}{a_n}U(s)+\frac{1}{s}\left(\frac{b_{n-1}}{a_n}U(s)-\frac{a_{n-1}}{a_n}Y(s)+\ldots+\frac{1}{s}\left(\frac{b_1}{a_n}U(s)-\frac{a_1}{a_n}Y(s)+\frac{1}{s}\left(\frac{b_0}{a_n}U(s)-\frac{a_0}{a_n}Y(s)\right)\underbrace{\ldots}_{n}\right)\right)$$